Test 2
Math 130
Fall 2016

$\frac{19}{30} = 63\%$

Name__ __________________________GUST #__

Time allowed 50 min.
You are allowed to use only **non scientific** calculators. You are NOT allowed to use phones.
Each question is 5 points.

1) Evaluate dy/dt for the function at the point. y'

$x^3 + y^3 = 9$; $dx/dt = -3$, $x = 1$, $y = 2$

$x' = -3$; $x = 1$, $y = 2$

$3x^2 \cdot x' + 3y^2 \cdot y' = 0$

$3 \cdot (1)^2 \cdot (-3) + 3 \cdot (2)^2 \cdot y' = 0$

$-9 + 12y' = 0$

$\frac{12y'}{12} = \frac{9}{12} \longrightarrow \boxed{y' = \frac{3}{4}}$ ✓

5

2) Determine the intervals where f(x) is concave up and the intervals where f(x) is concave down, find the inflection points for $f(x) = 3x^4 - 6x^2 + 7$.

f''

$x = \frac{1}{3}$ $x = 0$

$f' = 12x^3 - 12x$

$f'' = 36x^2 - 12$; $f'' = 0$

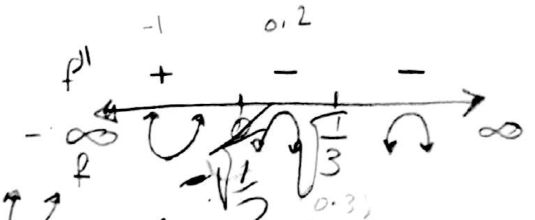
$36x^2 - 12 = 0$

$36x^2 = 12 \longrightarrow x^2 = \frac{1}{3} \longrightarrow x = \sqrt{\frac{1}{3}}$

$x\,(x - \frac{1}{3}) = 0$

$x = 0$ $x = \frac{1}{3}$ critical values

??



$\therefore f(x)$ is $\cup$ on $(-\infty, 0)$

$f(x)$ is $\cap$ on $(0, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

Inflection Points:

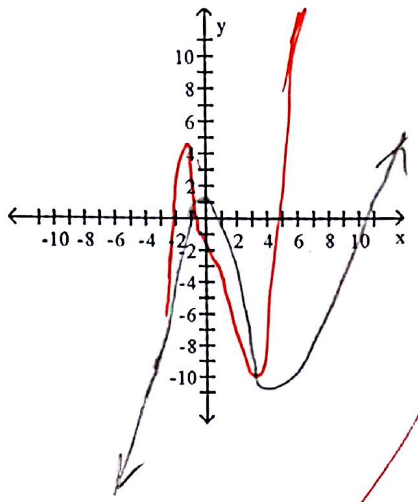
$f(0) = 7$

$f(-\sqrt{\frac{1}{3}}) = \dots$

$f(\sqrt{\frac{1}{3}}) = \dots$

2.5 P.

3) Find the intervals where the function increases, decreases, find the absolute max/min and sketch the graph of $f(x) = x^3 - 8x - 2$. Include the y-intercept (DO NOT FIND the x- intercepts)



$f' = 3x^2 - 8$; $f' = 0$

$3x^2 - 8 = 0$

$\frac{3x^2}{3} = \frac{8}{3}$

$x^2 = \frac{8}{3} \rightarrow \boxed{x = \pm \frac{2\sqrt{6}}{3}}$

$\boxed{\begin{array}{l} x = 1.63 \\ x = -1.63 \end{array}}$

f': + − +

$-\infty \nearrow -1.63 \searrow 1.63 \nearrow \infty$

f

$\therefore f(x) \nearrow$ on $(-\infty, -\frac{2\sqrt{6}}{3}) \cup (\frac{2\sqrt{6}}{3}, \infty)$

$f(x) \searrow$ on $(-\frac{2\sqrt{6}}{3}, \frac{2\sqrt{6}}{3})$

$f(-\frac{2\sqrt{6}}{3}) = -10.71$

$f(\frac{2\sqrt{6}}{3}) = 6.71$

4.5

Find the limit, if it exists.

4) Find $\lim_{x \to 0^+} \frac{e^x - 1}{x^2}$.

Form $\frac{0}{0} \Rightarrow$ L'Hop. rule

$\lim_{x \to 0^+} \frac{e^x - \cancel{1}}{2x^{\cancel{2}}} = \cancel{\lim_{x \to 0^+} \frac{(2x)\cdot(e^x - 1) - (e^x \cdot x^2)}{(x^2)^2}}$

$$\begin{array}{c|c} T & B \\ e^x - 1 & x^2 \\ \hline e^x & 2x \end{array}$$

NB

$\cancel{\lim_{x \to 0^+} \frac{2x \cdot e^x - 2x - e^x - x^2}{x^4}}$

NO

Form $\frac{1}{0^+} = \infty$

2 p.

6.5

5) The annual revenue and cost functions for a manufacturer of zip drives are approximately $\boxed{R(x) = 520x - 0.02x^2}$ and $C(x) = 160x + 100{,}000$, where x denotes the number of drives made. What is the maximum annual profit?

$P_{\text{Profit}} = R - C$

$= 520X - 0.02X^2 - (160X + 100{,}000)$

$P = 520X - 0.02X^2 - 160X - 100{,}000$

$P = 360 \overset{2}{\underset{x}{\;}} - 0.02X^2 - 100{,}000$ ✗ ?

$\boxed{P' = -0.04X}$; $P'' = -0.04$; $P'' = 0$; $-0.04 \neq 0$

$-0.04X = 0$ ✗

$\boxed{X = 0.04}$

2p.

P' : $-\infty$ ———— 0.04 ———— ∞

6) Find the absolute minimum value of $f(x) = 4x + x^2 + 2$ on $[0, \infty)$.

$f' = 4 + 2X$; $f' = 0$

$4 + 2X = 0$

$2X = -4 \rightarrow \boxed{X = -2}$ critical (open)

f' : $-\infty$ — -2 + ∞

~~$f'(-2) = 0$~~

$f(-2) = -2 \rightarrow$ abs. Minim.

$f(x) \downarrow$ on $(-\infty, -2)$

$f(x) \uparrow$ on $(-2, \infty)$

3p.

$f(0) = 2$

abs. min.

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