

Assignment 2

1. At a local university, a sample of 16 evening students was selected in order to determine whether the average age of the evening students is significantly different from 21. The average age of the students in the sample was 19 with a standard deviation of 3.5

a. state the null & alternative hypotheses for this test

$$H_0 : \mu = 21$$

$$H_1 : \mu \neq 21$$

b. find the test statistic

$$(t\text{-stat}) = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{19 - 21}{\frac{3.5}{\sqrt{16}}} = - 2.286$$

c. develop your critical region for the test at a 5% significance level

**Two sided test:  $\alpha/2 = 2.5\%$ , t- critical value (df = 15) =  $\pm 2.1315$**

d. what is your conclusion at the 5% significance level

**Since t-statistic falls in the rejection region, so we reject  $H_0$  at  $\alpha = 5\%$ ,**

e. find the P-value for this test, and what is your conclusion

$$P\text{-value} = 2(1\% - 2.5\%) = 2\% - 5\%$$

**At  $\alpha = 5\%$ , P-value <  $\alpha$ , so we reject  $H_0$  at  $\alpha = 5\%$**

2. Samples of final examination scores for two statistics classes with different instructors provided the following results.

Instructor A	Instructor B
$n_1 = 9$	$n_2 = 16$
$\bar{x}_1 = 72$	$\bar{x}_2 = 76$
$\sigma_1 = 8$	$\sigma_2 = 10$

a. develop a 95% confidence interval for the difference between the average grade for all the students at both professors' classes ( $\mu_A - \mu_B$ )

$$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, Z_{0.025} = 1.96$$

$$\begin{aligned} \mu_1 - \mu_2 &= (72 - 76) \pm 1.96 \sqrt{\frac{64}{9} + \frac{100}{16}} \\ &= -4 \pm 7.16 = (-11.16, 3.16) \end{aligned}$$

b. to test a claim that Instructor A is tougher than Instructor B (average grade is lower), state the null & alternative hypotheses for this test

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 < 0$$

c. find the test statistic for your test in part b

$$Z\text{-statistic} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{72 - 76}{\sqrt{\frac{64}{9} + \frac{100}{16}}} = -1.09$$

d. develop your critical region for your test in part b, at a 5% significance level

$$\text{left sided test: } Z\text{-critical} = -1.65$$

e. what is your conclusion at a 5% significance level for your test in part b

Since Z-statistic falls in the acceptance region, so we accept  $H_0$  at  $\alpha = 5\%$

f. find the P-value for your test in part b, and what is your conclusion

$$p\text{-value} = 0.5 - P(0 < Z < 1.09)$$

$$= 0.5 - 0.3621$$

$$= 0.1379$$

$$= 13.8\%$$

$$P\text{-value} > \alpha = 10\%$$

So we accept  $H_0$  at  $\alpha = 10\%$

g. summarize in a table the 2 good decisions vs. the 2 bad decisions, showing type I and II errors

Decision	$H_0$ true	$H_0$ false
accept $H_0$	correct decision	type II error
reject $H_0$	type I error	correct decision

3. A sample of 500 musicians from Las Vegas shows that the average age is 40 years, and 20% of the sample is female. Create a 99% interval for the proportion of all female musicians in Las Vegas

$$P = \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \text{where: } \hat{p} = 0.2, Z_{0.005} = 2.58$$

$$\text{So: } P = 0.2 \pm 2.58 \sqrt{\frac{0.2(0.8)}{500}}$$

$$= 0.2 \pm 0.05 = (0.15, 0.25)$$